Grade 7/8 Math Circles

February 13/14/15/16, 2023

Prime Time - Problem Set

1. State whether each of the following numbers are prime or composite. If composite, determine the amount of unique prime factors:
   (a) 63
   (b) 114
   (c) 47
   (d) 243
2. How many positive factors does 9690 have?
3. What fraction of integers between 1 and 30, inclusive, is prime?
4. The *greatest common divisor* (GCD) of two numbers a, b is the largest number that divides into both a and b. List the GCD for all the pairs of even numbers between 20 and 30.
5. What is the difference between the two greatest prime factors of 585?
6. Determine the smallest integer with exactly five unique factors.
7. The seven-digit number $6,227,\underline{d}32$ is divisible by 11. What is the digit d?
8. What are the possible k values for the four-digit number $561k$ if its prime factorization must include:
   (a) 3
   (b) 2
   (c) At least two powers of 3
   (d) 3 **and** 7
9. The three digit number $3a8$ is added to 243 and gives $6b1$. If $6b1$ is divisible by 9, find the value of $a \times b$.
10. The product of three different positive integers is 168. What is the largest possible sum of these three integers? *(Note: the integers must be greater than 1)*
11. Mr. Math has a box of protractors with a volume of 858cm^3. What are the possible dimensions of the box?
    (Recall: Volume = length × width × height)